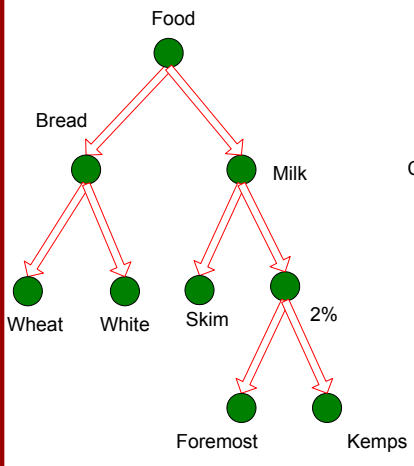
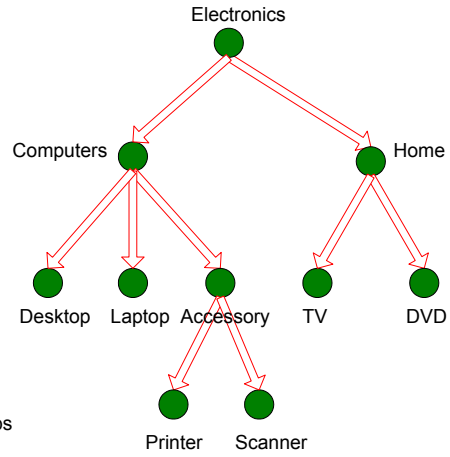
Knowledge Discovery and Data Mining

Unit # 16

Multi-level Association Rules





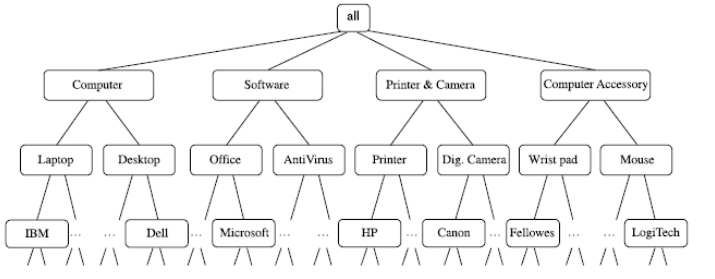
Multi-level Association Rules

- Why should we incorporate concept hierarchy?
  - Rules at lower levels may not have enough support to appear in any frequent itemsets
  - Rules at lower levels of the hierarchy are overly specific
    - e.g., skim milk → white bread, 2% milk → wheat bread, skim milk → wheat bread, etc.

      are indicative of association between milk and bread

Example

TID	Items Purchased
T100	IBM-ThinkPad-T40/2373, HP-Photosmart-7660
T200	Microsoft-Office-Professional-2003, Microsoft-Plus!-Digital-Media
T300	Logitech-MX700-Cordless-Mouse, Fellowes-Wrist-Rest
T400	Dell-Dimension-XPS, Canon-PowerShot-S400
T500	IBM-ThinkPad-R40/P4M, Symantec-Norton-Antivirus-2003
…	…

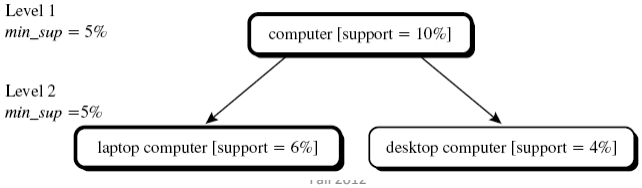


Concept Hierarchies

- Multilevel association rules can be mined efficiently using concept hierarchies under a support-confidence framework.
- In general, a top-down strategy is employed, where counts are accumulated for the calculation of frequent itemsets at each concept level, starting at the concept level 1 and working downward in the hierarchy toward the more specific concept levels, until no more frequent itemsets can be found.
- For each level, any algorithm for discovering frequent itemsets may be used, such as Apriori or its variations.

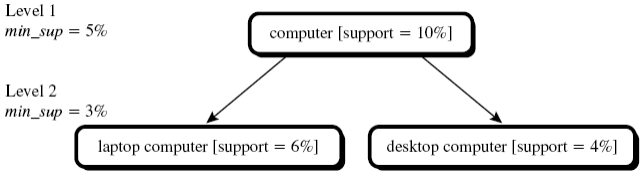
Uniform Minimum Support for All Levels

- The same minimum support threshold is used when mining at each level of abstraction.
- For example, a minimum support threshold of 5% is used throughout (e.g., for mining from "computer" down to "laptop computer").
- Both "computer" and "laptop computer" are found to be frequent, while "desktop computer" is not.



Reduced Minimum Support at Lower Levels

- Each level of abstraction has its own minimum support threshold. The deeper the level of abstraction, the smaller the corresponding threshold is.
- For example, the minimum support thresholds for levels 1 and 2 are 5% and 3%,respectively.
- In this way, "computer," "laptop computer," and "desktop computer" are all considered frequent.



Continuous and Categorical Attributes

How to apply association analysis formulation to non-asymmetric binary variables?

Session Id	Country	Session Length (sec)	Number of Web Pages viewed	Gender	Browser Type	Buy
1	USA	982	8	Male	IE	No
2	China	811	10	Female	Netscape	No
3	USA	2125	45	Female	Mozilla	Yes
4	Germany	596	4	Male	IE	Yes
5	Australia	123	9	Male	Mozilla	No
…	…	…	…	…	…	…

Example of Association Rule:

$\{\text{Number of Pages} \in [5,10) \wedge (\text{Browser=Mozilla})\} \rightarrow \{\text{Buy = No}\}$

Handling Categorical Attributes

- Transform categorical attribute into asymmetric binary variables.
- Introduce a new "item" for each distinct attribute-value pair
  - Example: replace Browser Type attribute with
    - Browser Type = Internet Explorer
    - Browser Type = Mozilla
    - Browser Type = Netscape
- What if attribute has many possible values
  - Example: attribute country has more than 200 possible values
  - Many of the attribute values may have very low support
    - Potential solution: Aggregate the low-support attribute values

Handling Continuous Attributes

- Different kinds of rules:
  - $\text{Age} \in [21,35) \wedge \text{Salary} \in [70k,120k) \rightarrow \text{Buy}$
  - $\text{Salary} \in [70k,120k) \wedge \text{Buy} \rightarrow \text{Age: } \mu=28, \sigma=4$
- Different methods:
  - Discretization-based
  - Statistics-based
  - Non-discretization based
    - minApriori

Discretization

- Discretization is the most common approach for handling continuous attributes.
- This approach groups the adjacent values of a continuous attribute into a finite number of intervals.
- The discrete intervals are then mapped into asymmetric binary attributes so that existing association analysis algorithms can be applied.

Discretization Issues

- Size of the discretized intervals affect support & confidence

  {Refund = No, (Income = $51,250)} $\rightarrow$ {Cheat = No}

  {Refund = No, (60K $\leq$ Income $\leq$ 80K)} $\rightarrow$ {Cheat = No}

  {Refund = No, (0K $\leq$ Income $\leq$ 1B)} $\rightarrow$ {Cheat = No}

  - If intervals too small
    - may not have enough support
  - If intervals too large
    - may not have enough confidence

Statistics-based Methods

- Example:

  Browser=Mozilla $\wedge$ Buy=Yes $\rightarrow$ Age: μ=23
- Rule consequent consists of a continuous variable, characterized by their statistics
  - mean, median, standard deviation, etc.
- Approach:
  - Withhold the target variable from the rest of the data
  - Apply existing frequent itemset generation on the rest of the data
  - For each frequent itemset, compute the descriptive statistics for the corresponding target variable
    - Frequent itemset becomes a rule by introducing the target variable as rule consequent
  - Apply statistical test to determine interestingness of the rule

Statistics-based Methods

- How to determine whether an association rule interesting?
  - Compare the statistics for segment of population covered by the rule vs segment of population not covered by the rule:

    A $\Rightarrow$ B: μ versus A $\Rightarrow$ B: μ'
  - Statistical hypothesis testing:
    - Null hypothesis: H0: $\mu' = \mu + \Delta$
    - Alternative hypothesis: H1: $\mu' > \mu + \Delta$
    - Z has zero mean and variance 1 under null hypothesis

$$Z = \frac{\mu' - \mu - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

Statistics-based Methods

- Example:

  r: Browser=Mozilla $\wedge$ Buy=Yes $\rightarrow$ Age: μ=23

  - Rule is interesting if difference between μ and μ' is greater than 5 years (i.e., Δ = 5)
  - For r, suppose n1 = 50, s1 = 3.5
  - For r' (complement): n2 = 250, s2 = 6.5

$$Z = \frac{\mu' - \mu - \Delta}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{30 - 23 - 5}{\sqrt{\frac{3.5^2}{50} + \frac{6.5^2}{250}}} = 3.11$$

  - For 1-sided test at 95% confidence level, critical Z-value for rejecting null hypothesis is 1.64.
  - Since Z is greater than 1.64, r is an interesting rule

Non-discretization Methods

- There are certain applications in which analysts are more interested in finding associations among the continuous attributes, rather than associations among discrete intervals of the continuous attributes.

Min-Apriori (Han et al)

Document-term matrix:

TID	W1	W2	W3	W4	W5
D1	2	2	0	0	1
D2	0	0	1	2	2
D3	2	3	0	0	0
D4	0	0	1	0	1
D5	1	1	1	0	2

Example:

W1 and W2 tends to appear together in the same document

Min-Apriori

- Data contains only continuous attributes of the same "type"
  - e.g., frequency of words in a document

TID	W1	W2	W3	W4	W5
D1	2	2	0	0	1
D2	0	0	1	2	2
D3	2	3	0	0	0
D4	0	0	1	0	1
D5	1	1	1	0	2

- Potential solution:
  - Convert into 0/1 matrix and then apply existing algorithms
    - lose word frequency information
  - Discretization does not apply as users want association among words not ranges of words

Min-Apriori

- How to determine the support of a word?
  - If we simply sum up its frequency, support count will be greater than total number of documents!
    - Normalize the word vectors – e.g., using L_1 norm
    - Each word has a support equals to 1.0

TID	W1	W2	W3	W4	W5
D1	2	2	0	0	1
D2	0	0	1	2	2
D3	2	3	0	0	0
D4	0	0	1	0	1
D5	1	1	1	0	2

Normalize

TID	W1	W2	W3	W4	W5
D1	0.40	0.33	0.00	0.00	0.17
D2	0.00	0.00	0.33	1.00	0.33
D3	0.40	0.50	0.00	0.00	0.00
D4	0.00	0.00	0.33	0.00	0.17
D5	0.20	0.17	0.33	0.00	0.33

Min-Apriori

- New definition of support:

$$\sup(C) = \sum_{i \in T} \min_{j \in C} D(i, j)$$

TID	W1	W2	W3	W4	W5
D1	0.40	0.33	0.00	0.00	0.17
D2	0.00	0.00	0.33	1.00	0.33
D3	0.40	0.50	0.00	0.00	0.00
D4	0.00	0.00	0.33	0.00	0.17
D5	0.20	0.17	0.33	0.00	0.33

Example:

Sup(W1,W2,W3)

= 0 + 0 + 0 + 0 + 0.17

= 0.17

Anti-monotone property of Support

TID	W1	W2	W3	W4	W5
D1	0.40	0.33	0.00	0.00	0.17
D2	0.00	0.00	0.33	1.00	0.33
D3	0.40	0.50	0.00	0.00	0.00
D4	0.00	0.00	0.33	0.00	0.17
D5	0.20	0.17	0.33	0.00	0.33

Example:

Sup(W1) = 0.4 + 0 + 0.4 + 0 + 0.2 = 1

Sup(W1, W2) = 0.33 + 0 + 0.4 + 0 + 0.17 = 0.9

Sup(W1, W2, W3) = 0 + 0 + 0 + 0 + 0.17 = 0.17